Name:____________________________

Date:_____________________________

Projectile Motion Vectors

There are multiple ways to represent an object's motion. If the motion is two-dimensional and lies in a plane, some representations include:

(1) recording *x* and *y* coordinates of the object at different times in a data table;

(2) displaying the object's *x* and *y* locations at regular time intervals on a diagram;

(3) drawing vectors showing displacement, velocity, and acceleration and their *x* and *y* components at different times.

(4) using vector equations to represent velocity and acceleration vectors quantitatively.

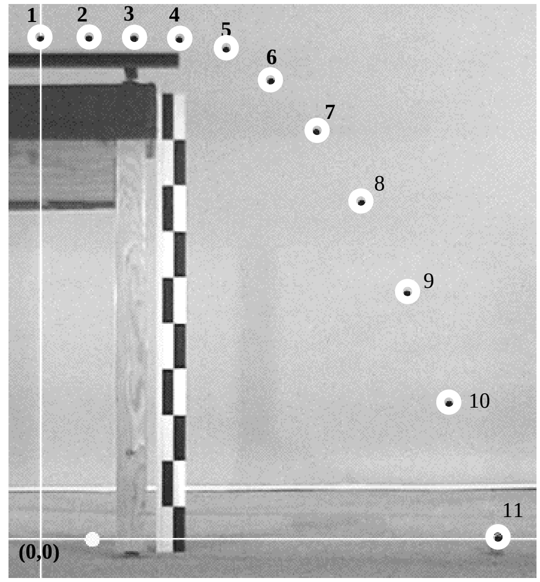


Figure 1: A motion diagram showing a ball's locations every 1/15ths as it rolls horizontally and then falls vertically for about 1 meter.

In this activity you will practice representing the motion shown in Figure 1 using vectors and vector equations that represent displacements as well as average velocities and accelerations in the 1/15th of a second time intervals between position measurements.

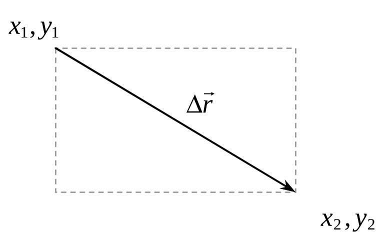
Before working on this activity, you should view the movie entitled <Galileo's Projectile_15fps.mov> and review the definitions of two-dimensional displacement and velocity vectors.

1. Preliminary Questions

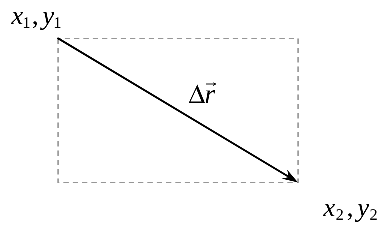
(a) Suppose an object is moving in a plane and we choose to describe it using an *x*-*y* coordinate system. If the object is at location x_1, y_1 at time t_1 and at location x_2, y_2 at time t_2, then it has been "displaced." The mathematical definition of its displacement vector is

$$\Delta\vec{r} = \Delta\vec{x} + \Delta\vec{y} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$

where $\Delta\vec{x}$ and $\Delta\vec{y}$ are the *x* and *y*-components of the object's displacement in the time period. A displacement vector $\Delta\vec{r}$ is shown in the diagram below. Draw and label the *x*-component of the displacement vector (denote it as $\Delta\vec{x}$) of the vector $\Delta\vec{r}$. Place the tail of the vector at x_1, y_1.
Hint: This vector component points in the *x*-direction only.



(b) Once again the displacement vector $\Delta\vec{r}$ is shown in the diagram below. Draw and label the *y*-component of the displacement vector (denote it as $\Delta\vec{y}$) of the vector $\Delta\vec{r}$. Place the tail of the vector at x_1, y_1. **Hints**: This component points in the *y*-direction only and the length of $\Delta\vec{y}$ is less than the length of $\Delta\vec{x}$.



(c) The vector equation that defines average velocity during a time interval $\Delta t = t_2 - t_1$ is

$$\langle \vec{v}_{1\to 2} \rangle = \frac{\Delta\vec{r}}{\Delta t} = \frac{(x_2 - x_1)}{(t_2 - t_1)}\hat{\imath} + \frac{(y_2 - y_1)}{(t_2 - t_1)}\hat{\jmath}.$$

Explain why the relative lengths of the displacement-vector components you drew in parts (a) and (b) should be proportional to the lengths of the corresponding velocity vector components.

(d) The motion diagram in Figure 1 is a composite diagram constructed from a video analysis of the movie <Galileo's Projectile_15fps.mov>. The figure shows the position of the falling ball at equally spaced time intervals of $1/15^{th}$ of a second (with $t_2 – t_1 = t_3 – t_2 = t_4 – t_3$ and so on). Explain why the relative lengths of the displacement vector components should also be proportional to the lengths of the corresponding velocity vector components that represent the average velocity during each time interval.

2. Activity-Based Questions

(a) **Construct Vectors with Lengths Proportional to the Horizontal and Vertical Velocities**: Open the Logger *Pro* experiment file <ProjectileVectors.cmbl> containing a video analysis file of the movie <Galileo's Projectile_15fps.mov>. The movie has been scaled in meters. If you view the movie on a frame-by-frame basis for all 11 frames, you will see that video analysis has already been used to determine the location of the ball in each frame.

The images that follow are motion diagrams created based on a video analysis of the ball's path. Use the fact that the lengths of displacement and velocity vectors are proportional to each other to draw a series of vectors that are proportional to the average *x* and *y*-velocity vector components during each 1/15th of a second time interval. Start with Frame 1 in each of the figures that follow. Place the tail of the first velocity vector at the ball's location in frame 1 and then place the tail of the next vector at the ball's location in frame 2 and so on.

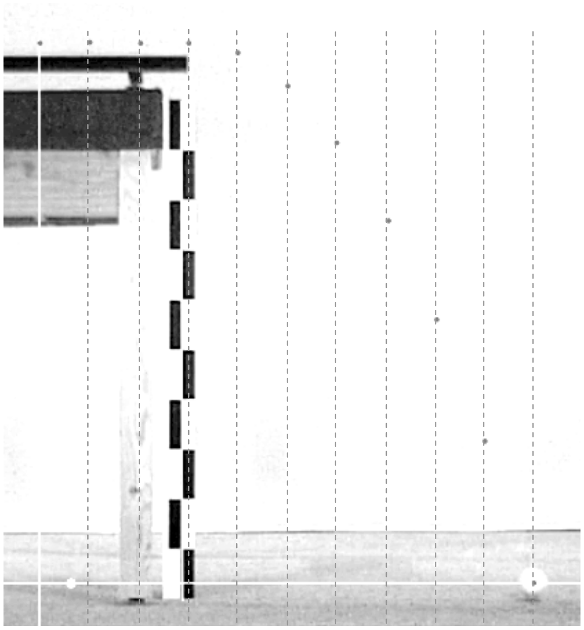


Figure 2: *x*-components of velocity

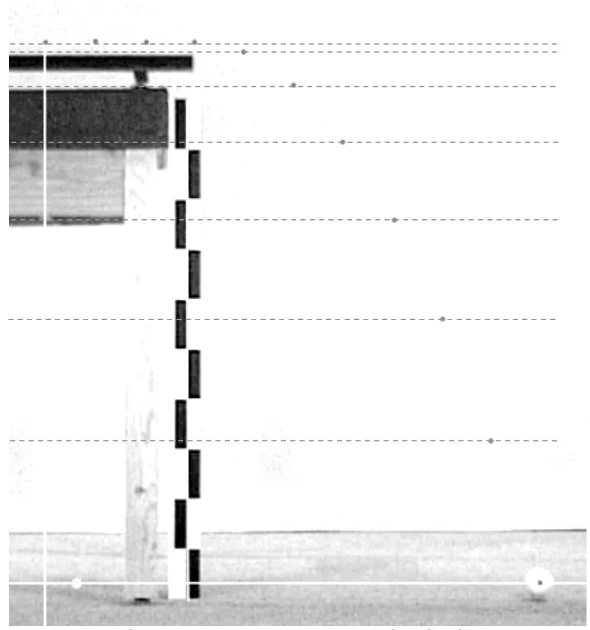


Figure 3: *y*-components of velocity

(b) What happens to the *x*-component of the ball's velocity, v_x, as the ball moves horizontally? Does it decrease, increase or remain that same? What happens to the v_x vectors? Explain how you arrived at your answers.

(c) In the first three frames of the movie, what happens to the *y*-component of velocity of the ball? Does the ball's vertical velocity component decrease, increase, or remain that same? What happens to the v_y vectors? Explain how you arrived at your answer.

(d) In the last eight frames of the movie, what happens to the *y*-component of velocity of the ball? Does the ball's vertical velocity component decrease, increase, or remain that same? What happens to the v_y vectors? Explain how you arrived at your answers.

3. Reflections on Your Findings:

(a) **What do changes in velocity vector components tell us about acceleration?** Begin this reflection by writing the definition for the average acceleration during a time interval $\Delta t = t_2 - t_1$ in terms of the horizontal and vertical velocity vector components using $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ notation.

(b) What can you conclude about the nature of the horizontal acceleration of this ball based on your results in section 2(a)?

(c) What can you conclude about the nature of vertical acceleration for this ball in frames 1 through 3? Explain.

(d) What can you conclude about the nature of vertical acceleration for this ball in frames 4 through 11? Explain your reasoning carefully. **Hint**: If you take a millimeter ruler and measure the relative length of the vertical velocity vector in frames 4 through 11, approximately how much does the length of the y-component of average velocity between frames change from frame to frame? Is the direction of change positive or negative? Is there a positive or negative acceleration or no acceleration?